Student Name (**Print**) | Student Signature

THE UNIVERSITY OF NEW BRUNSWICK

Calculus Challenge Exam

Tuesday, June 4, 2019 **6:00-9:00pm**

INSTRUCTIONS

1. **Do not unstaple the exam or detach any pages.**
2. **Questions begin on Page 2 and continue to Page 12. Check that you have a complete exam.**
3. **The exam is closed book/closed notes. No notes or books or extra paper (other than that provided) are permitted.**
4. **Phones/tablets/smartwatches and other communication devices must be left at the front of the examination room. Accessing such devices during the examination shall be viewed as an academic offence.**
5. **No calculators, translators or electronic aides of any type are permitted.**
6. **The back of each page is blank and may be used for rough work.**
7. **Show all your work and justify your answers. Show all work in the space provided.**
8. **The final exam grade will be converted to a letter grade based on the following conversion scale:**

$A+$: [92, 100]	$B+$: [75,80)	$C+$: [58,65)
A : [86,92)	B : [70,75)	C : [50,58)
$A-$: [80,86)	$B-$: [65,70)	D : [45,50)
		F : [0,45)

A letter grade of $B-$ or higher must be achieved on the exam to qualify for credit for Math 1003.

FOR GRADING ONLY

Page	MARK
2	/8
3	/8
4	/6
5	/20
6	/10
7	/10
8	/14
9	/9
10-12	/15
TOTAL	/100

LETTER GRADE

The examination was graded by

Professor Name (**Print**) | Professor Signature

[22mks] **Part I Short Answer Questions: Answer each question with a short explanation or calculation.**

1. Find the domain of the function $f(x) = \dfrac{\sqrt{x}}{x^2 - 4}$. Write your answer in interval notation.

2. Find all values of x in the interval $[0, 2\pi]$ that satisfy

$$\sin(2x) = \cos(x)$$

3. Suppose $f(x) = 2\cos^{-1}(x)$. What is the value of $f(1)$?

4. Simplify as much as possible: $\dfrac{\ln(32) - \ln(8)}{\ln(8)}$

5. Find the indicated limits. If the limit is infinite, write $+\infty$ or $-\infty$. If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

(a) Find the value of $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x-1}$.

(b) Find the value of $\displaystyle\lim_{x\to 0} \frac{\tan(6x)}{\sin(5x)}$.

(c) Find the value of $\displaystyle\lim_{x\to 0} \frac{2e^x-x}{x^2}$.

6. Show that $f(x) = x - \cos(x)$ has a zero (that is, $f(x) = 0$ has a solution) in the interval $\left[0, \frac{\pi}{2}\right]$. Justify your answer by stating what theorem you used.

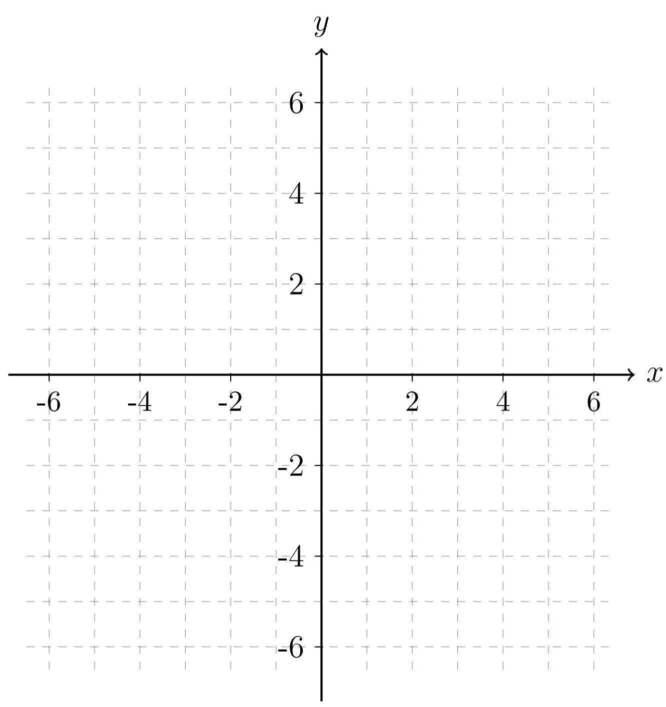
7. A certain radioactive isotope has a half-life of 500 years. How much of a 100 mg sample will remains after 1500 years? (You do not need to simplify your answer.)

8. Suppose f is differentiable everywhere, $f'(5) = 0$ and $f''(5) = 2$. Which of the following statements must be true ?

(A) $f(5)$ is a local minimum.
(B) $f(5)$ is a local maximum.
(C) $f(5)$ is the absolute minimum.
(D) $f(5)$ is the absolute maximum.
(E) f is decreasing on $(4, 6)$.
(F) none of the above.

9. Sketch a graph of a function $f(x)$ such that:

- $f(x)$ has a removable discontinuity at $x = -4$,
- $\lim_{x\to 0^-} f(x) = 4$,
- $\lim_{x\to 0^+} f(x) = -4$,
- $f(x)$ is right continuous at $x = 0$, and
- $\lim_{x\to 4^-} f(x) = \infty$.



Part II: Long Answer Questions. Show all work in the space provided. Full marks will be awarded only for complete and justified solutions.

[20mks] **1.** Find $\dfrac{dy}{dx}$. Do not simplify your answers.

(a) $y = \ln\left(2 - x^3\right) + \dfrac{1}{5x^4} + e^{3x}$

(b) $y = \sqrt{\dfrac{\cos(x)}{2\log_3(x-1)}}$

(c) $y = 4x\tan^{-1}\left(\sqrt{x}\right)$

(d) $y = \sin^2(x^2) - 3^{2x}$

(e) $y = \cosh(5x) + 5\sec\left(\dfrac{1}{x}\right)$

[4mks] **2.** Find $\dfrac{dy}{dx}$ if

$$\sin(y^3 - 1) - 5xy = x^3 + 6.$$

[6mks] **3.** Consider $f(x) = \sqrt{x-1}$.

(a) Use the **limit definition** of the derivative to find $f'(x)$.

(b) Find an equation of the line tangent to the curve $f(x)$ at the point $(5, 2)$.

[3mks] **4.** Suppose f is continuous on $[2,5]$ and $-3 \leq f'(x) \leq 4$ for all values of x in $(2,5)$. Show that $-9 \leq f(5) - f(2) \leq 12$. State what theorem you use.

[4mks] **5.** Find all horizontal asymptote(s) of $f(x) = \dfrac{2e^x}{e^x - 5}$.

[3mks] **6.** Find the value of $\lim\limits_{x \to \infty} xe^{-x^2}$.

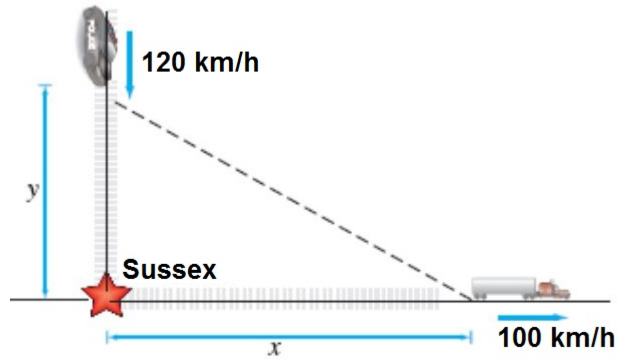
[5mks] **7.** Find the absolute maximum and minimum values of $f(x) = \dfrac{\ln(x)}{x}$ on the interval $[1, e^2]$.

[4mks] **8.** Use logarithmic differentiation to find $\dfrac{dy}{dx}$ where $y = (3x-1)^{\sqrt{x}}$.

[5mks] **9.** Find the point on the line $y = 2x + 3$ that is closest (has minimal distance) to the origin.

[5mks] **10.** A police car traveling south toward Sussex at 120 km/h pursues a truck traveling east away from Sussex, at 100 km/h (see diagram). At time $t = 0$, the police car is 40 km north and the truck is 60 km east of Sussex.

Calculate the rate at which the distance between the vehicles is changing 15 minutes later.



[4mks] **11.** **(a)** Find the general antiderivative of $f'(x) = 2x - \sin x$.

(b) Solve the initial value problem:

$$\frac{dy}{dx} = 2x - \sin x \quad , \quad y(0) = 5$$

[15mks] **12.** Suppose $f(x) = \dfrac{x^2}{x-2}$, $f'(x) = \dfrac{x(x-4)}{(x-2)^2}$, and $f''(x) = \dfrac{8}{(x-2)^3}$.

This question spans three pages. Find the following information. **Justify your answers/show your work**. Use the information to sketch the graph of $f(x)$ on the axes provided.

(a) Domain of $f(x)$.

(b) Symmetry of $f(x)$. (Determine whether f is even, odd, or neither.)

(c) Any intercept(s).

(d) Any horizontal asymptotes. Justify your answer.

(e) Any vertical asymptotes. Justify your answer.

(f) Critical numbers.

(g) Open intervals of increase/decrease.

(h) Local extrema (provide points) if any. If there are none, say so.

(i) Open interval(s) where $f(x)$ is concave up/down.

(j) Inflection points, if any. If there are none, say so.

(k) Sketch: