Last name: ______________________ **First name:** ______________________

Middle initial(s): ________

Date of birth: ______________________

High school: ______________________ **Teacher:** ______________________

I would like to be considered for credit at (circle one):

UNB-Fredericton **UNB-Saint John**

(Note: If offered credit, you will have the choice to accept or decline.)

UNIVERSITY OF NEW BRUNSWICK
DEPARTMENT OF MATHEMATICS & STATISTICS

Calculus Challenge Exam
Wednesday, June 6, 2018

Time: 3 hours (no calculators, notes or electronics).

Page	**Points**	**Mark**
3	20	
4	12	
5	7	
6	14	
7	11	
8	7	
9	7	
10	7	
11-12	15	
Total:	100	

Instructions

1. This exam has 13 pages (including this cover page) and is **printed double-sided**.
2. Do each question in the indicated space. Pages 2 and 13 are intentionally left blank in case you need additional space. If you do, **clearly indicate** where your answers are located. **Please do not separate pages of the exam.**
3. Show your work! Full marks are awarded only when the answer is correct and supported with reasons.
4. Calculators, cell phones, laptops, tablets, and all other electronic devices are **not** permitted. Notes, books, scrap paper and aids of any kind are forbidden.

EXTRA PAGE – do not remove this page

MARKS

[20] 1. Find y'. Answers **need not** be simplified.

(a) $y = \left(\dfrac{1-x}{1+x}\right)^3$

(b) $y = \arctan(x^2) + \tan(\sqrt{x})$

(c) $y = \sin^2(x) - \log_3(4x)$

(d) $y = \sinh(e^{2x})$

(e) $y = 4^{4x\cos x}$

MARKS

[12] 2. Find the indicated limits. If the limit is infinite, write "$+\infty$" or "$-\infty$". If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

(a) $\displaystyle \lim_{x \to -\infty} \frac{\sqrt{x + 16x^2}}{x + 1}$

(b) $\displaystyle \lim_{x \to 2} \frac{2 - x}{2x^2 - 8}$

(c) $\displaystyle \lim_{h \to 0} \frac{\sqrt{64 + h} - 8}{h}$

(d) $\displaystyle \lim_{x \to \infty} x \sin\left(\frac{\pi}{x}\right)$

[7] 3. Use the limit definition of the derivative to compute the derivative of $f(x) = \frac{2}{x}$.

4. Consider the curve described by the equation

$$2xy - \pi \cos(y) = 3\pi.$$

[5] (a) Find $\frac{dy}{dx}$ (in terms of x and y).

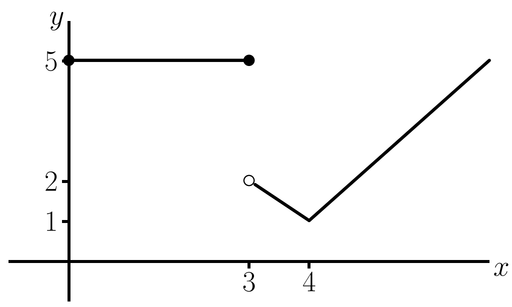
[3] (b) Find an equation of the line tangent to the curve at the point $(1, 2\pi)$.

[6] 5. Find $\frac{dy}{dx}$ when

$$y = (3x - 1)^{\sqrt{x}}$$

MARKS

[5] 6. Referring to the function f, shown in the graph, circle **all** correct statements.



(a) $\lim_{x\to 3^+} f(x)$ does not exist.

(b) $\lim_{x\to 3} f(x)$ does not exist.

(c) $f(3)$ does not exist.

(d) f is not continuous at 4.

(e) f is not differentiable at 4.

[6] 7. Find the absolute maximum and the absolute minimum of the function given by

$$f(x) = 2x^3 - 9x^2 + 12x$$

on the interval $[0, 3]$.

MARKS

[7] 8. A potato is put into a 200°C oven. The temperature of the potato as a function of time is modeled by

$$T = a(1 - e^{-kt}) + b$$

where t is time in minutes and a, b, and k are constants.

(a) Given that the potato starts at 20°C, and that the temperature of the potato will approach the temperature of the oven over time, find a and b. Hint: use limits.

(b) Given that the temperature of the potato initially increases at a rate of 2°C per minute, find k.

(c) How long will it take the potato to reach 110°C? (Leave your answer in exact form.)

[7] 9. A room measures 30 feet by 17 feet. You are purchasing a rug for the room that must cover 200 square feet. The cost of the rug is \$5 per square foot, plus \$1 per foot for the edging. Thus, you are looking to minimize the amount of edging. Find the dimensions of the cheapest rug that will cover the required area. Use calculus to justify your answer, and write a complete solution showing your steps.

[7] 10. A water balloon (which you may assume is spherical) has a 15 centimetre diameter. Water begins leaking out at a rate of 8 millilitres per second. How fast is the diameter of the balloon decreasing when the diameter is 4 centimetres? (Recall that 1mL = 1 cm^3 and that the volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

MARKS

[15] 11. This question extends over two pages. Answer all parts of this question with regard to the graph of $y = f(x)$, where $f(x) = \dfrac{x^2}{x^2 - 4}$

(a) State the domain of f.

(b) Determine any vertical or horizontal asymptotes. Justify your answers using limits.

(c) Find the intervals on which f is increasing, and the intervals on which it is decreasing.

(d) Find the coordinates of any local/relative extrema.

(e) Find the intervals on which the graph is concave up, and the intervals on which it is concave down.

(f) Find the coordinates of any points of inflection.

(g) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (e) above.

EXTRA PAGE – do not remove this page